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A Mixture of Viscous Elastic Materials with Different Constituent Temperatures

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Abstract. Constitutive equations are discussed for a mixture of any number of materials with elastic and viscous properties in which the constituents may have different temperatures.

1. Introduction

The recent literature on the continuum theory of mixtures contains rather diverse points of view in the primitive concepts and the forms of the basic field equations. The basic equations representing balances of masses, linear momenta, moments of momenta, and energy for each constituent together with equations for the mixture as a whole obtained by summation, as developed by Truesdell [1,2], Truesdell and Toupin [3] and a number of other writers are closely related and are based on similar points of view. On the other hand, the development of the theory by Green and Naghdi [4] and Mills [5] is based on different primitive concepts and consequently some of their resulting equations have different forms from those in [1,3]. However, in a recent paper confined to mixtures with a single temperature, the differences and the relationship between the two approaches have been clarified by Green and Naghdi [6]:

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The basic equations in the two developments are equivalent, although this is not apparent at first sight in the case of some of the equations* -- the differences lie chiefly in the primitive concepts, the premise under which the two forms of the theory are developed, in addition to the interpretations associated with some of the quantities which occur in the equations. For further detail regarding the relationship between the two forms of the theory, as well as additional references on the subject, we refer the reader to [6].

In the present paper, we are concerned with a mixture of any number of constituents each of which is at a different temperature. Since we now regard the previous paper of Green and Naghdi [7] involving many temperatures as too restrictive, we first reconsider here the forms of the basic equations, as well as the entropy inequality for the mixture as a whole. Although the discussion of the basic equations in sections 3-5 involves some repetition of known results, it is included here in order to have the basic equations in line with our point of view [6] and in terms of variables which we believe provide simple interpretations when discussing constitutive equations.

In section 6, we first briefly outline two procedures for the discussion of constitutive equations and adopt the simpler of the two which is slightly more restrictive than the other. We then consider a mixture of ν constituents (with different temperatures) which are not necessarily gases and which have viscous and elastic properties. In particular, we

*In [6], Green and Naghdi have given a different interpretation to the internal energy in the energy equation used by them in [4] and have recast this equation in a slightly different form.

obtain explicit results for the "equilibrium" values of various quantities and also include a discussion of the results when all constituents have the same temperature.

2. Notation

We summarize relevant notation for kinematic quantities and density. We consider a mixture of ν interacting constituents, each of which is regarded as a continuum; we refer to the α^{th} constituent as the continuum s_α $(\alpha = 1,2,..,\nu)$. We assume that each point within the mixture is occupied simultaneously by all ν constituents, which are in motion relative to a fixed system of rectangular Cartesian axes. The position of a typical particle of s_α at time τ is denoted by $x_i^\alpha(\tau)$, where

$$x_i^\alpha(\tau) = x_i^\alpha(X_1^\alpha,\ X_2^\alpha,\ X_3^\alpha,\ \tau) \qquad (-\infty < \tau \leqq t) \quad , \tag{2.1}$$

and X_i^α is a reference position of the particles of s_α . All Latin indices are tensor indices and take the values 1,2,3, and the usual summation convention is employed. The Greek letter α , as in (2.1), is reserved for reference to the α^{th} constituent of the mixture only. Unless indicated otherwise the summation convention does not apply to Greek indices. We use the notation

$$x_i^\alpha = x_i^\alpha(t) \quad , \tag{2.2}$$

assume that the particles of s_α $(\alpha = 1,2,..,\nu)$ all occupy the same position at time t so that

$$x_i^1 = x_i^2 = \quad = x_i^\nu = x_i \quad , \tag{2.3}$$

and we refer to this position at time t as x_i .

The velocity vectors at the point x_i in s_α at time t are given by

$$v_i^\alpha = \frac{\overset{\alpha}{D}\, x_i^\alpha}{Dt} \qquad (\alpha = 1,2,..,\nu) \quad , \tag{2.4}$$

where $\overset{\alpha}{D}/Dt$ denotes differentiation with respect to t holding X_k^α fixed in continuum s_α . This operator may be written in the form

$$\frac{\overset{\alpha}{D}}{Dt} = \frac{\partial}{\partial t} + v_k^\alpha \frac{\partial}{\partial x_k} \quad . \tag{2.5}$$

Acceleration vectors at time t are

$$f_i^\alpha = \frac{\overset{\alpha}{D}\, v_i^\alpha}{Dt} = \frac{\partial v_i^\alpha}{\partial t} + v_k^\alpha \frac{\partial v_i^\alpha}{\partial x_k} \qquad (\alpha = 1,2,..,\nu) \quad , \tag{2.6}$$

the densities of s_α at time t are ρ_α and the rates of deformation and vorticity tensors for s_α at time t are given by

$$2d_{ik}^\alpha = v_{i,k}^\alpha + v_{k,i}^\alpha \quad , \quad 2\omega_{ik}^\alpha = v_{i,k}^\alpha - v_{k,i}^\alpha \quad , \tag{2.7}$$

where a comma stands for partial differentiation with respect to x_i .

We define total density ρ and mean velocity v_i by the expressions

$$\rho = \sum_{\alpha=1}^{\nu} \rho_\alpha \quad , \tag{2.8}$$

$$\rho v_i = \sum_{\alpha=1}^{\nu} \rho_\alpha v_i^\alpha \quad , \tag{2.9}$$

and put

$$u_i^{\alpha} = v_i^{\alpha} - v_i \quad , \tag{2.10}$$

so that

$$\sum_{\alpha=1}^{\nu} \rho_{\alpha} u_i^{\alpha} = 0 \quad . \tag{2.11}$$

We also define the operator

$$(\dot{\ }) = \frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + v_k \frac{\partial(\ )}{\partial x_k} \quad , \tag{2.12}$$

and observe that

$$\frac{\overset{\alpha}{D}}{Dt} = \frac{D}{Dt} + \sum_{\beta=1}^{\nu} \frac{\rho_{\beta}}{\rho} (v_k^{\alpha} - v_k^{\beta}) \frac{\partial}{\partial x_k} = \frac{D}{Dt} + u_k^{\alpha} \frac{\partial}{\partial x_k} \quad . \tag{2.13}$$

3. Mass and momentum

Using a fixed surface A enclosing an arbitrary volume V the equation of mass balance for s_α is[+]

$$\frac{\partial}{\partial t}\int_V \rho_\alpha \, dV + \int_A \rho_\alpha v_k^\alpha n_k \, dA = \int_V m_\alpha \, dV \quad , \tag{3.1}$$

where m_α is density of mass production arising from all the other constituents and n_k is the unit outward normal to A . Hence

$$m_\alpha = \frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial x_k}(\rho_\alpha v_k^\alpha) = \frac{\overset{\alpha}{D} \rho_\alpha}{Dt} + \rho_\alpha d_{kk}^\alpha \quad , \tag{3.2}$$

and we add the condition

$$\sum_{\alpha=1}^{\nu} m_\alpha = 0 \quad . \tag{3.3}$$

Next we consider the equation of linear momentum for s_α in the form

[+]Alternatively we could use material volume elements of each constituent which coincide at time t .

$$\frac{\partial}{\partial t}\int_V \rho_\alpha v_i^\alpha \, dV + \int_A \rho_\alpha v_i^\alpha v_k^\alpha n_k \, dA - \int_V m_\alpha \hat{v}_i^\alpha \, dV$$

$$= \int_V (\rho_\alpha F_i^\alpha + p_i^\alpha) dV + \int_A n_k \sigma_{ki}^\alpha \, dA \quad , \tag{3.4}$$

where F_i^α is external body force applied to s_α and σ_{ki}^α is the stress tensor[++] associated with the ingredient s_α so that the total force acting on s_α across A per unit area is $n_k \sigma_{ki}^\alpha$. The term involving $\hat{v}_i^\alpha$ is similar to that used by Ingram and Eringen [8] and allows for the fact that momentum is supplied to s_α not from rest but from all the other constituents of the mixture with some mean velocity $\hat{v}_i^\alpha$ which will depend on all the velocities of the remaining constituents, but it is not necessary to make any special assumption about this. The vector p_i^α may now be called a diffusive force acting on s_α. In addition to (3.4) we impose the condition

$$\sum_{\alpha=1}^{\nu} (p_i^\alpha + m_\alpha \hat{v}_i^\alpha) = 0 \quad . \tag{3.5}$$

In point form equation (3.4) becomes

$$\sigma_{ki,k}^\alpha + \rho_\alpha (F_i^\alpha - f_i^\alpha) = m_\alpha v_i^\alpha - p_i^\alpha - m_\alpha \hat{v}_i^\alpha \quad , \tag{3.6}$$

and we assume that p_i^α is a vector unaltered by superposed rigid body motions of the whole mixture, apart from orientation. For convenience we say that p_i^α is objective but other writers give this word a slightly

[++]Truesdell and Toupin [3] use a different order for the indices on the stress tensor which, in general, is not symmetric.

different meaning. Professor Chadwick has suggested that (3.6) be put in a more convenient form

$$\sigma^{\alpha}_{ki,k} + \rho_{\alpha}(F^{\alpha}_{i} - f^{\alpha}_{i}) = \mu^{\alpha}_{i} + m_{\alpha}\, u^{\alpha}_{i} \quad , \tag{3.7}$$

where μ^{α}_{i} is objective and

$$m_{\alpha}\, v^{\alpha}_{i} - I^{\alpha}_{i} - m_{\alpha}\, \hat{v}^{\alpha}_{i} = \mu^{\alpha}_{i} + m_{\alpha}\, u^{\alpha}_{i} \quad , \quad \sum_{\alpha=1}^{\nu} \mu^{\alpha}_{i} = 0 \quad . \tag{3.8}$$

From (3.7) and (3.8) we recover the equations of motion for the mixture as a whole. Thus

$$\sigma_{ki,k} + \sum_{\alpha=1}^{\nu} \rho_{\alpha}(F^{\alpha}_{i} - f^{\alpha}_{i}) - \sum_{\alpha=1}^{\nu} m_{\alpha}\, v^{\alpha}_{i} = 0 \quad , \tag{3.9}$$

where

$$\sigma_{ki} = \sum_{\alpha=1}^{\nu} \sigma^{\alpha}_{ki} \quad . \tag{3.10}$$

Corresponding to (3.4) we postulate a moment of momentum equation

$$e_{ijk}\, L_{ij} = 0 \quad , \tag{3.11}$$

where e_{ijk} is the alternating tensor and

$$L_{ij} = \frac{\partial}{\partial t}\int_V \rho \overset{\alpha}{v}_i x_j \, dV + \int_A \rho_\alpha \overset{\alpha}{v}_i \overset{\alpha}{v}_k x_j n_k \, dA - \int_V m_\alpha \hat{\overset{\alpha}{v}}_i x_j \, dV$$

$$- \int_V (\rho_\alpha \overset{\alpha}{F}_i + \overset{\alpha}{p}_i) x_j \, dV - \int_A n_k \overset{\alpha}{\sigma}_{ki} x_j \, dA - \int_V \overset{\alpha}{\lambda}_{ji} \, dV \quad . \tag{3.12}$$

In (3.12) $\overset{\alpha}{\lambda}_{ji}$ is a skew symmetric tensor which may be called the diffusive couple acting on s_α. With the help of (3.6), equation (3.11) yields

$$\overset{\alpha}{\sigma}_{[ki]} = - \overset{\alpha}{\lambda}_{ki} \quad , \tag{3.13}$$

where $\overset{\alpha}{\sigma}_{[ki]}$ is the skew symmetric part of $\overset{\alpha}{\sigma}_{ki}$. We add the condition

$$\sum_{\alpha=1}^{\nu} \overset{\alpha}{\lambda}_{ki} = 0 \quad , \tag{3.14}$$

so that

$$\sigma_{[ki]} = 0 \quad . \tag{3.15}$$

4. Energy balance and entropy production for mixture

Before discussing energy balance equations for each constituent s_α we obtain the energy equation for the mixture. We assume that

$$\begin{aligned}
&\frac{\partial}{\partial t}\int_V \sum_{\alpha=1}^{\nu}\left(\rho_\alpha U_\alpha + \frac{1}{2}\rho_\alpha v_i^\alpha v_i^\alpha\right)dV \\
&\quad + \int_A \sum_{\alpha=1}^{\nu}\left(\rho_\alpha v_k^\alpha U_\alpha + \frac{1}{2}\rho_\alpha v_k^\alpha v_i^\alpha v_i^\alpha\right)n_k \, dA \\
&\quad = \int_V\left(\rho r + \sum_{\alpha=1}^{\nu}\rho_\alpha F_i^\alpha v_i^\alpha\right)dV + \int_A\left(\sum_{\alpha=1}^{\nu} t_i^\alpha v_i^\alpha - h\right)dA \quad ,
\end{aligned} \tag{4.1}$$

where we take

$$t_i^\alpha = n_k \sigma_{ki}^\alpha \quad , \quad h = n_k q_k \quad , \quad q_k = \sum_{\alpha=1}^{\nu} q_k^\alpha \quad , \quad \rho r = \sum_{\alpha=1}^{\nu}\rho_\alpha r_\alpha \quad , \tag{4.2}$$

q_k^α being the heat flux vector for s_α and q_k the heat flux vector for the mixture, r_α is heat supply per unit mass of s_α, per unit time, including the heat supply arising from all other constituents, with r the total heat supply per unit mass of the mixture; the internal heat supplies will be specified by constitutive equations but do not affect the value of r in $(4.2)_4$. Also U_α is internal energy of s_α per unit mass, allowing for interaction between s_α and the other constituents. With the help of (3.7) the point form of (4.1) becomes

$$\begin{aligned}
&\rho r - q_{k,k} - \sum_{\alpha=1}^{\nu}\left(\rho_\alpha \frac{\overset{\alpha}{D} U_\alpha}{Dt} + m_\alpha U_\alpha\right) \\
&\quad + \sum_{\alpha=1}^{\nu}\left(\mu_i^\alpha v_i^\alpha + \sigma_{ki}^\alpha v_{i,k}^\alpha + \frac{1}{2} m_\alpha u_i^\alpha u_i^\alpha\right) = 0 \quad .
\end{aligned} \tag{4.3}$$

Since

$$\sum_{\alpha=1}^{\nu} \left(\rho_\alpha \frac{\overset{\alpha}{D} U_\alpha}{Dt} + m_\alpha U_\alpha\right) = \rho \dot{U} + \Phi \quad , \tag{4.4}$$

where

$$\rho U = \sum_{\alpha=1}^{\nu} \rho_\alpha U_\alpha \quad , \quad \Phi = \sum_{\alpha=1}^{\nu} \frac{\partial}{\partial x_k} \left(\rho_\alpha u_k^\alpha U_\alpha\right) \quad , \tag{4.5}$$

equation (4.3) has the alternative form

$$\rho r - q_{k,k} - \rho \dot{U} - \Phi + \sum_{\alpha=1}^{\nu} \left(\mu_i^\alpha v_i^\alpha + \sigma_{ki}^\alpha v_{i,k}^\alpha + \frac{1}{2} m_\alpha u_i^\alpha u_i^\alpha\right) \quad . \tag{4.6}$$

We adopt an entropy inequality for the mixture which is equivalent to one used by Truesdell [2], namely

$$\frac{\partial}{\partial t} \int_V \sum_{\alpha=1}^{\nu} \rho_\alpha S_\alpha \, dV + \int_A \sum_{\alpha=1}^{\nu} \rho_\alpha v_k^\alpha S_\alpha n_k \, dA$$
$$- \int_V \sum_{\alpha=1}^{\nu} \frac{\rho_\alpha r_\alpha}{T_\alpha} dV + \int_A \sum_{\alpha=1}^{\nu} \frac{q_k^\alpha}{T_\alpha} n_k \, dA \geqq 0 \quad , \tag{4.7}$$

where T_α (> 0) is the temperature of the constituent s_α and S_α is its entropy per unit mass. In point form (4.7) becomes

$$\sum_{\alpha=1}^{\nu} \left(\rho_\alpha \frac{\overset{\alpha}{D} S_\alpha}{Dt} + m_\alpha S_\alpha\right) - \sum_{\alpha=1}^{\nu} \frac{\rho_\alpha r_\alpha}{T_\alpha} + \sum_{\alpha=1}^{\nu} \left(\frac{q_k^\alpha}{T_\alpha}\right)_{,k} \geqq 0 \quad . \tag{4.8}$$

Except when all the temperatures are equal we cannot develop the consequence of the inequality (4.8) with the help only of (4.3). We need energy equations for each constituent so as to be able to eliminate r_α from (4.8) and also to provide sufficient equations for the temperatures T_α $(\alpha = 1,..,\nu)$. We consider this in the next section.

5. Energy balance for each constituent

Recalling quantities defined in previous sections we postulate an energy balance for s_α in the form

$$\frac{\partial}{\partial t}\int_V \rho_\alpha(U_\alpha + \frac{1}{2} v_i^\alpha v_i^\alpha)dV + \int_A \rho_\alpha(U_\alpha + \frac{1}{2} v_i^\alpha v_i^\alpha)v_k^\alpha n_k \, dA - \int_V \frac{1}{2} m_\alpha \hat{v}_i^\alpha \hat{v}_i^\alpha \, dV$$

$$= \int_V \{\rho_\alpha(r_\alpha + F_i^\alpha v_i^\alpha) + p_i^\alpha v_i^\alpha + \lambda_{ki}^\alpha \omega_{ik}^\alpha\} \, dV$$

$$+ \int_A (t_i^\alpha v_i^\alpha - n_k q_k^\alpha)dA + \int_V \psi_\alpha \, dV - \int_A n_k \bar{q}_k^\alpha \, dA \quad . \tag{5.1}$$

The scalar ψ_α represents a volume contribution to the energy equation arising from interaction, in addition to interactions already considered, and $n_k \bar{q}_k^\alpha$ represents an interaction surface flow of energy with $\bar{q}_k^\alpha$ an interaction flux vector. However, in point form this vector contributes $\bar{q}_{k,k}^\alpha$ which can be absorbed into ψ_α, so without loss of generality we set $\bar{q}_k^\alpha = 0$. The constituent s_α has a mass increase acquired from the other constituents and the rate of change of its kinetic energy just before becoming a part of s_α is represented by the third group of terms in (5.1). All other quantities in (5.1) have been defined previously and follow naturally from the discussion of sections 3,4.

Equation (5.1) can be reduced to

$$\int_V \{ \rho_\alpha \frac{\overset{\alpha}{D}\, U_\alpha}{Dt} + m_\alpha\, U_\alpha + \rho_\alpha (f_i^\alpha - F_i^\alpha) v_i^\alpha + \frac{1}{2}\, m_\alpha\, v_i^\alpha\, v_i^\alpha - \frac{1}{2}\, m_\alpha\, \hat{v}_i^\alpha\, \hat{v}_i^\alpha \}\, dV$$

$$= \int_V (\rho_\alpha r_\alpha + p_i^\alpha\, v_i^\alpha + \lambda_{ki}^\alpha\, \omega_{ik}^\alpha + \psi_\alpha) dV + \int_V (t_i^\alpha\, v_i^\alpha - n_k\, q_k^\alpha) dA \quad . \tag{5.2}$$

If

$$\sum_{\alpha=1}^{\nu} (\psi_\alpha + \frac{1}{2}\, m_\alpha\, \hat{v}_i^\alpha\, \hat{v}_i^\alpha + p_i^\alpha\, v_i^\alpha + \lambda_{ki}^\alpha\, \omega_{ik}^\alpha) = 0 \quad , \tag{5.3}$$

and if we also use (4.2), then by summation of equations (5.2) we find that

$$\int_V \sum_{\alpha=1}^{\nu} \{ \rho_\alpha \frac{\overset{\alpha}{D}\, U_\alpha}{Dt} + m_\alpha U_\alpha - \rho_\alpha r_\alpha + \rho_\alpha (f_i^\alpha - F_i^\alpha) v_i^\alpha + \frac{1}{2}\, m_\alpha\, v_i^\alpha\, v_i^\alpha \}\, dV$$

$$= \int_A (\sum_{\alpha=1}^{\nu} t_i^\alpha\, v_i^\alpha - n_k q_k) dA \quad . \tag{5.4}$$

Equation (5.4) is the same as (4.1) and with the help of (3.7) we recover the point form (4.3).

Using (3.8), as well as (3.13) and (3.3), equation (5.3) reduces to

$$\sum_{\alpha=1}^{\nu} (\mu_i^\alpha\, v_i^\alpha + \overset{\alpha}{\sigma}_{[ki]} \overset{\alpha}{\omega}_{ik} + \frac{1}{2}\, m_\alpha\, u_i^\alpha\, u_i^\alpha - \Psi_\alpha) = 0 \quad , \tag{5.5}$$

where

$$\Psi_\alpha = \psi_\alpha + \frac{1}{2}\, m_\alpha (\hat{v}_i^\alpha - v_i^\alpha)\, (\hat{v}_i^\alpha - v_i) \quad . \tag{5.6}$$

Also, with the help of (3.7), (3.13) and (5.6), the point form of (5.2)

becomes

$$\rho_\alpha r_\alpha - \overset{\alpha}{q}_{k,k} - \rho_\alpha \frac{\overset{\alpha}{D} U_\alpha}{Dt} - m_\alpha U_\alpha + \overset{\alpha}{\sigma}_{(ki)} \overset{\alpha}{d}_{ik} + \Psi_\alpha = 0 \quad , \tag{5.7}$$

where $\overset{\alpha}{\sigma}_{(ki)}$ is the symmetric part of $\overset{\alpha}{\sigma}_{ki}$. The arbitrary functions Ψ_α are subject to the condition (5.5). It is desirable to simplify this condition and one way to achieve this is to put+

$$\Psi_\alpha = \overset{\alpha}{\mu}_i(\overset{\alpha}{v}_i - \overset{\nu}{v}_i) + \overset{\alpha}{\sigma}_{[ki]}(\overset{\alpha}{\omega}_{ik} - \overset{\nu}{\omega}_{ik}) + \frac{1}{2} m_\alpha \overset{\alpha}{u}_i \overset{\alpha}{u}_i + \Theta_\alpha \quad , \tag{5.8}$$

so that

$$\sum_{\alpha=1}^{\nu} \Theta_\alpha = 0 \quad , \tag{5.9}$$

and (5.7) becomes

$$\rho_\alpha r_\alpha - \overset{\alpha}{q}_{k,k} - \rho_\alpha \frac{\overset{\alpha}{D} U_\alpha}{Dt} - m_\alpha U_\alpha + \frac{1}{2} m_\alpha \overset{\alpha}{u}_i \overset{\alpha}{u}_i + \Theta_\alpha$$

$$+ \overset{\alpha}{\mu}_i(\overset{\alpha}{v}_i - \overset{\nu}{v}_i) + \overset{\alpha}{\sigma}_{(ki)} \overset{\alpha}{d}_{ik} + \overset{\alpha}{\sigma}_{[ki]}(\overset{\alpha}{\omega}_{ik} - \overset{\nu}{\omega}_{ik}) = 0 \quad . \tag{5.10}$$

We observe that for some purposes it is useful to replace the ν equations (5.10) by an equivalent set, namely equation (4.3) and the $\nu - 1$ equations

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+This is not the only possible form for Ψ_α. For example, the first terms in (5.8) could be replaced by $\overset{\alpha}{\mu}_i (\overset{\alpha}{v}_i - v_i)$, the difference being absorbed in Θ_α, but (5.8) is more convenient for our purpose.

$$\rho_\alpha r_\alpha - \rho_\nu r_\nu - \overset{\alpha}{q}_{k,k} + \overset{\nu}{q}_{k,k} - \rho_\alpha \frac{\overset{\alpha}{D} U_\alpha}{Dt} + \rho_\nu \frac{\overset{\nu}{D} U_\nu}{Dt} - m_\alpha U_\alpha + m_\nu U_\nu$$

$$+ \frac{1}{2} m_\alpha \overset{\alpha}{u}_i \overset{\alpha}{u}_i - \frac{1}{2} m_\nu \overset{\nu}{u}_i \overset{\nu}{u}_i + \Theta_\alpha - \Theta_\nu + \overset{\alpha}{\mu}_i(\overset{\alpha}{v}_i - \overset{\nu}{v}_i) + \overset{\alpha}{\sigma}_{(ki)} \overset{\alpha}{d}_{ik}$$

$$- \overset{\nu}{\sigma}_{(ki)} \overset{\nu}{d}_{ik} + \overset{\alpha}{\sigma}_{[ki]}(\overset{\alpha}{\omega}_{ik} - \overset{\nu}{\omega}_{ik}) = 0 \quad . \tag{5.11}$$

In equation (5.10) the functions U_α , Θ_α , $\overset{\alpha}{\mu}_i$, $\overset{\alpha}{\sigma}_{ki}$, $\overset{\alpha}{q}_k$, m_α are all objective.

If we eliminate r_α from (4.8) with the help of (5.10) we obtain the inequality

$$- \sum_{\alpha=1}^{\nu} \frac{1}{T_\alpha} \{\rho_\alpha(\frac{\overset{\alpha}{D} A_\alpha}{Dt} + S_\alpha \frac{\overset{\alpha}{D} T_\alpha}{Dt}) + m_\alpha A_\alpha - \frac{1}{2} m_\alpha \overset{\alpha}{u}_i \overset{\alpha}{u}_i - \Theta_\alpha\}$$

$$+ \sum_{\alpha=1}^{\nu} \frac{1}{T_\alpha} \{\overset{\alpha}{\mu}_i(\overset{\alpha}{v}_i - \overset{\nu}{v}_i) + \overset{\alpha}{\sigma}_{(ki)} \overset{\alpha}{d}_{ik} + \overset{\alpha}{\sigma}_{[ki]}(\overset{\alpha}{\omega}_{ik} - \overset{\nu}{\omega}_{ik})\} - \sum_{\alpha=1}^{\nu} \frac{\overset{\alpha}{q}_k T_{\alpha,k}}{T_\alpha^2} \geq 0 \quad , \tag{5.12}$$

where

$$A_\alpha = U_\alpha - T_\alpha S_\alpha \quad . \tag{5.13}$$

The inequality (5.12) may be written in an alternative form which is useful for some purposes. Since

$$\sum_{\alpha=1}^{\nu} \left\{ \frac{\rho_\alpha}{T_\alpha} \left(\frac{\overset{\alpha}{D} A_\alpha}{Dt} + S_\alpha \frac{\overset{\alpha}{D} T_\alpha}{Dt} \right) + \frac{m_\alpha A_\alpha}{T_\alpha} \right.$$

$$= \sum_{\alpha=1}^{\nu} \left\{ \rho_\alpha \frac{\overset{\alpha}{D}}{Dt} \left(\frac{A_\alpha}{T_\alpha} \right) + \frac{m_\alpha A_\alpha}{T_\alpha} + \frac{\rho_\alpha}{T_\alpha} \left(S_\alpha + \frac{A_\alpha}{T_\alpha} \right) \frac{\overset{\alpha}{D} T_\alpha}{Dt} \right\}$$

$$= \rho \frac{DA}{Dt} + \sum_{\alpha=1}^{\nu} \frac{\partial}{\partial x_k} \left(\frac{\rho_\alpha \overset{\alpha}{u}_k A_\alpha}{T_\alpha} \right) + \sum_{\alpha=1}^{\nu} \frac{\rho_\alpha S^*_\alpha}{T_\alpha} \left(\frac{DT_\alpha}{Dt} + \overset{\alpha}{u}_k \frac{\partial T_\alpha}{\partial x_k} \right) \quad , \qquad (5.14)$$

where

$$S^*_\alpha = S_\alpha + \frac{A_\alpha}{T_\alpha} \quad , \qquad \rho A = \sum_{\alpha=1}^{\nu} \frac{\rho_\alpha A_\alpha}{T_\alpha} \quad , \qquad (5.15)$$

the inequality (5.12) becomes

$$- \rho \frac{DA}{Dt} - \sum_{\alpha=1}^{\nu} \frac{\rho_\alpha S^*_\alpha}{T_\alpha} \frac{DT_\alpha}{Dt} + \sum_{\alpha=1}^{\nu} \frac{1}{T_\alpha} \left(\frac{1}{2} m_\alpha \overset{\alpha}{u}_i \overset{\alpha}{u}_i + \Theta_\alpha \right)$$

$$- \sum_{\alpha=1}^{\nu} \frac{\partial}{\partial x_k} \left(\frac{\rho_\alpha \overset{\alpha}{u}_k A_\alpha}{T_\alpha} \right) - \sum_{\alpha=1}^{\nu} \frac{\overset{*}{q}_k T_{\alpha,k}}{T^2_\alpha}$$

$$+ \sum_{\alpha=1}^{\nu} \frac{1}{T_\alpha} \{ \overset{\alpha}{\mu}_i (\overset{\alpha}{v}_i - \overset{\nu}{v}_i) + \overset{\alpha}{\sigma}_{(ki)} \overset{\alpha}{d}_{ik} + \overset{\alpha}{\sigma}_{[ki]} (\overset{\alpha}{\omega}_{ik} - \overset{\nu}{\omega}_{ik}) \} \geq 0 \quad , \qquad (5.16)$$

where

$$\overset{*\alpha}{q}_k = \overset{\alpha}{q}_k + \rho_\alpha T_\alpha \overset{\alpha}{u}_k S^*_\alpha \quad . \qquad (5.17)$$

We observe that

$$\sum_{\alpha=1}^{\nu} \frac{\rho_\alpha u_k^\alpha A_\alpha}{T_\alpha} = \sum_{\alpha=1}^{\nu} \phi_\alpha (v_k^\alpha - v_k^\nu) = \sum_{\alpha=1}^{\nu} \phi_\alpha v_k^\alpha \quad ,$$

$$\phi_\alpha = \rho_\alpha \left(\frac{A_\alpha}{T_\alpha} - A\right) \quad , \quad \sum_{\alpha=1}^{\nu} \phi_\alpha = 0 \quad . \tag{5.18}$$

Hence, if we put

$$\mu_i^\alpha = T_\alpha \frac{\partial \phi_\alpha}{\partial x_k} + \bar{\mu}_i^\alpha \quad (\alpha = 1,..,\nu-1) \quad , \quad \sigma_{ki}^\alpha = T_\alpha \phi_\alpha \delta_{ki} + \bar{\sigma}_{ki}^\alpha \quad (\alpha = 1,..,\nu), \tag{5.19}$$

the inequality (5.16) reduces to

$$- \rho \frac{DA}{Dt} - \sum_{\alpha=1}^{\nu} \frac{\rho_\alpha \overset{*}{S}_\alpha}{T_\alpha} \frac{DT_\alpha}{Dt} + \sum_{\alpha=1}^{\nu} \frac{1}{T_\alpha} \left(\frac{1}{2} m_\alpha u_i^\alpha u_i^\alpha + \Theta_\alpha\right)$$

$$+ \sum_{\alpha=1}^{\nu} \frac{1}{T_\alpha} \{\bar{\mu}_i^\alpha (v_i^\alpha - v_i^\nu) + \bar{\sigma}_{(ki)}^\alpha d_{ik}^\alpha + \bar{\sigma}_{[ki]}^\alpha (\omega_{ik}^\alpha - \omega_{ik}^\nu)\}$$

$$- \sum_{\alpha=1}^{\nu} \frac{\overset{*\alpha}{q_k} T_{\alpha,k}}{T_\alpha^2} \geqq 0 \quad , \tag{5.20}$$

and

$$\mu_i^\nu = - \sum_{\alpha=1}^{\nu-1} \mu_i^\alpha \quad , \quad \sum_{\alpha=1}^{\nu} \bar{\sigma}_{[ki]}^\alpha = 0 \quad . \tag{5.21}$$

For later convenience we introduce the notation

$$\theta_\alpha = \frac{1}{T_\alpha} \quad , \quad \Delta_\alpha = \theta_\alpha - \theta_\nu \quad , \quad V_k^\alpha = v_k^\alpha - v_k^\nu \quad , \quad \Gamma_{ik}^\alpha = \omega_{ik}^\alpha - \omega_{ik}^\nu \, , \tag{5.22}$$

so that (5.20) becomes

$$- \rho \frac{DA}{Dt} + \sum_{\alpha=1}^{\nu} \frac{\rho_\alpha \overset{*}{S}_\alpha}{\theta_\alpha} \frac{D\theta_\alpha}{Dt} + \frac{1}{2} \sum_{\alpha=1}^{\nu} \theta_\alpha m_\alpha u_i^\alpha u_i^\alpha + \sum_{\alpha=1}^{\nu-1} \Theta_\alpha \Delta_\alpha$$

$$+ \sum_{\alpha=1}^{\nu} (\theta_\nu + \Delta_\alpha) \{\bar{\mu}_i^\alpha V_i^\alpha + \bar{\sigma}_{(ki)}^\alpha d_{ik}^\alpha + \bar{\sigma}_{[ki]}^\alpha \Gamma_{ik}^\alpha\}$$

$$+ \sum_{\alpha=1}^{\nu} \overset{*\alpha}{q_k} \theta_{\alpha,k} \geqq 0 \quad . \qquad (5.23)$$

6. Constitutive equations

We assume that mass elements of each constituent are conserved so that

$$m_{\alpha} = 0 \quad , \tag{6.1}$$

and that the constituents of the mixture have elastic and viscous properties. We therefore assume that A_{α}, S_{α}, $\sigma^{\alpha}_{(ki)}$, q^{α}_{k} $(\alpha = 1,..,\nu)$, μ^{α}_{k}, $\sigma^{\alpha}_{[ki]}$, θ_{α} $(\alpha = 1,..,\nu-1)$, are functions of+

$$\begin{gathered} \theta_{\nu} \quad , \quad \Delta_{\gamma} \quad , \quad F^{\beta}_{ij} = \frac{\partial x^{\beta}_{i}}{\partial X^{\beta}_{j}} \quad , \quad G^{\beta}_{ijk} = \frac{\partial^2 x^{\beta}_{i}}{\partial X^{\beta}_{j}\, \partial X^{\beta}_{k}} \quad , \\ \theta_{\beta,k} \quad , \quad V^{\gamma}_{k} \quad , \end{gathered} \tag{6.2}$$

and

$$v^{\beta}_{i,k} \quad , \tag{6.3}$$

for $\beta = 1,..,\nu$, $\gamma = 1,..,\nu-1$. In view of invariance conditions under superposed rigid body motions of the whole mixture, the velocity gradients must be replaced by

$$d^{\beta}_{ik} \quad , \quad \Gamma^{\gamma}_{ik} \quad . \tag{6.4}$$

+These constitutive equations satisfy equipresence which we regard as a mathematical convenience and not a physical principle.

If we use the inequality (5.12) to obtain restrictions on these constitutive assumptions, it can be shown that A depends only on

$$T_\beta \quad , \quad F^\beta_{ij} \quad . \tag{6.5}$$

In addition, further restrictions are placed on A_α and other dependent functions, but there is some algebraic complexity in making explicit deductions about the form of A_α from these restrictions.

Inspection of (5.20) suggests an alternative procedure. We assume the A_α, S^*_α (and hence A, S, ϕ_α), $\bar{\sigma}^\alpha_{(ki)}$, $q^{*\alpha}_k$ ($\alpha = 1,..,\nu$), $\bar{\mu}^\alpha_k$, $\bar{\sigma}^\alpha_{[ki]}$, Θ_α ($\alpha = 1,..,\nu-1$) are functions of the variables (6.2) and (6.4). From the inequality (5.20) we again find that A depends only on the variables (6.5). From (5.17), (5.18) and (5.19) we see that q^α_k and σ^α_{ki} are functions of the variables (6.2) and (6.4), but that μ^α_k are functions of

$$\theta_{\beta,kr} \quad , \quad \frac{\partial G^\beta_{ijk}}{\partial X^\beta_r} \quad , \quad d^\beta_{ik,r} \quad , \quad \Gamma^\beta_{ik,r} \quad ,$$

as well as the variables (6.2) and (6.4). If we add the additional restriction that μ^α_k should depend only on the quantities (6.2) and (6.4), it follows that ϕ_α reduce to functions of the variables

$$\theta_\nu \quad , \quad \Delta_\beta \quad , \quad F^\beta_{ij} \quad , \quad V^\beta_k \quad . \tag{6.6}$$

Using $(5.18)_2$ and the fact that A is a function of the quantities (6.5), it follows that A_α reduce to functions of the variables (6.6).

Restrictions on the remaining constitutive equations may now be found from (5.12), (5.16), (5.20) or (5.23), and we use (5.23). This procedure appears to be slightly more restrictive than the first and is the one adopted here.

We adopt the notation

$$Y_A = \{\Delta_\alpha, \theta_{\alpha,k}, V_k^\alpha, d_{ik}^\alpha, \Gamma_{ik}^\alpha\} \qquad (A = 1,..,16\nu-7) \quad , \tag{6.7}$$

and we put

$$\begin{aligned}
\overset{\alpha}{\sigma}_{ki} &= {}_o\overset{\alpha}{\sigma}_{ki} + {}_e\overset{\alpha}{\sigma}_{ki} \quad ,\\
\overset{-\alpha}{\sigma}_{ki} &= {}_o\overset{-\alpha}{\sigma}_{ki} + {}_e\overset{-\alpha}{\sigma}_{ki} \quad ,\\
\overset{\alpha}{\mu}_{k} &= {}_o\overset{\alpha}{\mu}_{k} + {}_e\overset{\alpha}{\mu}_{k} \quad ,\\
\overset{-\alpha}{\mu}_{k} &= {}_o\overset{-\alpha}{\mu}_{k} + {}_e\overset{-\alpha}{\mu}_{k} \quad ,\\
\overset{\alpha}{q}_{k} &= {}_o\overset{\alpha}{q}_{k} + {}_e\overset{\alpha}{q}_{k} \quad ,\\
\overset{*\alpha}{q}_{k} &= {}_o\overset{*\alpha}{q}_{k} + {}_e\overset{*\alpha}{q}_{k} \quad ,\\
\Theta_\alpha &= {}_o\Theta_\alpha + {}_e\Theta_\alpha \quad ,
\end{aligned} \tag{6.8}$$

where the quantities with a prefix o are the values of the corresponding quantities on the left hand side of (6.8) when $Y_A = 0$. The remaining terms in (6.8) may be regarded as polynomials of degree greater than zero in the functions Y_A with coefficients which are functions of the remaining variables in (6.2).

With the help of (5.20) we have

$$\rho_\alpha S^*_\alpha = \rho\theta_\alpha \frac{\partial A}{\partial \theta_\alpha} = -\rho T_\alpha \frac{\partial A}{\partial T_\alpha} \quad , \tag{6.9}$$

so that

$$\rho_\alpha S_\alpha = \theta_\alpha \sum_{\beta=1}^{\nu} \rho_\beta \theta_\beta \frac{\partial A_\beta}{\partial \theta_\alpha} = -T_\alpha \sum_{\beta=1}^{\nu} \frac{\rho_\beta}{T_\beta} \frac{\partial A_\beta}{\partial T_\alpha} \quad . \tag{6.10}$$

Also, we write

$$A_\alpha = A'_\alpha + A''_\alpha \quad , \quad A = A' + A'' \quad , \; \rho A' = \theta_\nu \sum_{\alpha=1}^{\nu} \rho_\alpha A'_\alpha \quad , \tag{6.11}$$

where A'_α are the values of A_α when Δ_β , V^β_k vanish, and A' is the value of A when Δ_β vanishes. For the present we regard A''_α as a polynomial of degree greater than zero in Δ_β , V^β_k and A'' as a polynomial of degree greater than zero in Δ_β with coefficients which are functions of θ_ν , F^β_{ij} . The result $(6.11)_3$ follows from $(5.15)_2$. Again, using (5.23), we have

$$_0\bar{\sigma}^\alpha_{ki} = \frac{\rho}{\theta_\nu} \frac{\partial A'}{\partial F^\alpha_{ij}} F^\alpha_{kj} \quad , \tag{6.12}$$

$$_0\bar{\mu}^\alpha_k = \frac{1}{\theta_\nu} \sum_{\beta=1}^{\nu} \left(\rho_\alpha \frac{\partial A'}{\partial F^\beta_{ij}} G^\beta_{ijr} \frac{\partial X^\beta_r}{\partial x^\beta_k} - \rho_\beta \frac{\partial A'}{\partial F^\alpha_{ij}} G^\alpha_{ijr} \frac{\partial X^\alpha_r}{\partial x^\alpha_k} \right) \quad , \tag{6.13}$$

for $\alpha = 1,..,\nu$ provided

$$\sum_{\alpha=1}^{\nu} \left(\frac{\partial A'}{\partial F^{\alpha}_{ij}} F^{\alpha}_{kj} - \frac{\partial A'}{\partial F^{\alpha}_{kj}} F^{\alpha}_{ij} \right) = 0 \quad . \tag{6.14}$$

Also

$${}_{o}\hat{\epsilon}_{\alpha} = 0 \quad , \quad {}_{o}\overset{*\alpha}{q}_{k} = 0 \quad , \quad {}_{o}q^{\alpha}_{k} = 0 \quad . \tag{6.15}$$

In addition there is a residual inequality which we omit. From (5.18), (5.19), (6.12) and (6.13) we then have

$${}_{o}\sigma^{\alpha}_{ki} = \sum_{\beta=1}^{\nu} \rho_{\beta} \frac{\partial A'_{\beta}}{\partial F^{\alpha}_{ij}} F^{\alpha}_{kj} \quad , \tag{6.16}$$

$${}_{o}\mu^{\alpha}_{k} = \sum_{\beta=1}^{\nu} \left(\rho_{\alpha} \frac{\partial A'_{\alpha}}{\partial F^{\beta}_{ij}} G^{\beta}_{ijr} \frac{\partial X^{\beta}_{r}}{\partial x^{\beta}_{k}} - \rho_{\beta} \frac{\partial A'_{\beta}}{\partial F^{\alpha}_{ij}} G^{\alpha}_{ijr} \frac{\partial X^{\alpha}_{r}}{\partial x^{\alpha}_{k}} \right) \quad , \tag{6.17}$$

in agreement with previous results [9].

The above constitutive equations are subject to the usual invariance conditions under superposed rigid body motions of the whole mixture. For example, A_{α}, A reduce to the new forms

$$A_{\alpha} = A_{\alpha}\left(\theta_{\nu} \, , \, \Delta_{\gamma} \, , \, \frac{\partial x^{\alpha}_{r}}{\partial X^{\alpha}_{i}} \frac{\partial x^{\beta}_{r}}{\partial X^{\beta}_{j}} \, , \, \frac{\partial x^{\alpha}_{r}}{\partial X^{\alpha}_{i}} v^{\beta}_{r} \right) \quad , \tag{6.18}$$

$$A = A\left(\theta_{\nu} \, , \, \Delta_{\gamma} \, , \, \frac{\partial x^{\alpha}_{r}}{\partial X^{\alpha}_{i}} \frac{\partial x^{\beta}_{r}}{\partial X^{\beta}_{j}} \right) \quad , \tag{6.19}$$

for $\alpha, \beta = 1, .., \nu$, $\gamma = 1, .., \nu-1$. In view of (6.19) we may verify that the condition (6.14) is satisfied.

In obtaining the above constitutive equations, we have adopted the second procedure described earlier in this section [after (6.5)]. We

note that if we had used the first method, we would still recover the results (6.9), (6.12), (6.13), (6.16) and (6.17) with A'_α being the "equilibrium" values of A_α, i.e., the values of A_α when Y_A vanish.

The special case of the formulae (6.10) when all the temperatures are equal is of some interest. In order to see what happens to (6.10) we write

$$\theta_\nu = \Delta \quad , \quad \theta_\alpha - \theta_\nu = \Delta_\alpha \quad (\alpha = 1,..,\nu-1) \quad , \tag{6.20}$$

then

$$\begin{aligned} \frac{\partial}{\partial\theta_\alpha} &= \frac{\partial}{\partial\Delta_\alpha} \qquad (\alpha \neq \nu) \quad , \\ \frac{\partial}{\partial\theta_\nu} &= \frac{\partial}{\partial\Delta} - \sum_{\alpha=1}^{\nu-1} \frac{\partial}{\partial\Delta_\alpha} \quad , \end{aligned} \tag{6.21}$$

and

$$\sum_{\alpha=1}^{\nu} \theta_\alpha \frac{\partial}{\partial\theta_\alpha} = \Delta \frac{\partial}{\partial\Delta} + \sum_{\alpha=1}^{\nu-1} \Delta_\alpha \frac{\partial}{\partial\Delta_\alpha} \quad . \tag{6.22}$$

The functions A_β may be expressed in the form

$$A_\beta = A^*_\beta(F^\alpha_{ij}, V^\alpha_k, \Delta, \Delta_\gamma) \quad . \tag{6.23}$$

When

$$\Delta_\gamma = 0 \quad (\gamma = 1,..,\nu-1) \quad , \quad \Delta = 1/T \quad , \tag{6.24}$$

where T is the common temperature of the constituents, we assume that

$$A_\beta \to A^*_\beta(F^\alpha_{ij}, V^\alpha_k, T) \quad , \tag{6.25}$$

$$\frac{\partial A_\beta}{\partial \Delta} \to \frac{\partial A_\beta^*}{\partial \Delta} = -\frac{1}{\Delta^2}\frac{\partial A_\beta^*}{\partial T} \quad . \tag{6.26}$$

It will not, in general, be possible to evaluate $\partial A_\beta/\partial\Delta_\gamma$ as $\Delta_\gamma \to 0$ in terms of the function A_β^* and we assume that these derivatives are replaced by arbitrary finite scalar functions⁺. From (6.10) and (6.21) we see that under the condition (6.24),

$$\sum_{\alpha=1}^{\nu} \rho_\alpha S_\alpha = \sum_{\beta=1}^{\nu} \Delta^2 \rho_\beta \frac{\partial A_\beta^*}{\partial \Delta} = -\sum_{\beta=1}^{\nu} \rho_\beta \frac{\partial A_\beta^*}{\partial T} \quad , \tag{6.27}$$

or, defining

$$\rho S = \sum_{\alpha=1}^{\nu} \rho_\alpha S_\alpha \quad , \quad \rho A^* = \sum_{\beta=1}^{\nu} \rho_\beta A_\beta^* \quad , \tag{6.28}$$

then

$$S = -\frac{\partial A^*}{\partial T} \quad , \tag{6.29}$$

as given previously [6,8]. Also $S_\alpha - S_\nu$ become arbitrary functions of F_{ij}^β, V_k^β, T not expressible in terms of A_β^*, which again agrees with a previous result [9]. We recall that the energy equations, which provide ν differential equations for the ν temperatures T_α, may be taken to be (4.3) and the ν-1 equations (5.11), and we observe that volume supplies of heat may be taken to be given by the ν independent quantities

⁺This is analagous to the procedure used in the passage from a compressible to an incompressible elastic material.

$$\rho r = \sum_{\alpha=1}^{\nu} \rho_\alpha r_\alpha \quad , \quad \rho_\beta r_\beta - \rho_\nu r_\nu \qquad (\beta = 1,..,\nu-1) \quad . \tag{6.30}$$

When temperatures reduce to a single temperature we have one equation (4.3) for this. The remaining equations (5.11) can then, in general, only be satisfied by appropriate choices for the heat supplies $\rho_\beta r_\beta - \rho_\nu r_\nu$. This is similar to the situation in which we can, in general, only maintain isothermal conditions with T constant by an appropriate choice of r .

Special cases of the results (6.16) and (6.17) may be obtained without difficulty. For example, if constitutive equations depend on the deformation gradients F^β_{ij} only through the densities ρ_β , then

$${}_0\overset{\alpha}{\sigma}_{ki} = - \delta_{ki} \sum_{\beta=1}^{\nu} \rho_\alpha \rho_\beta \frac{\partial A'_\beta}{\partial \rho_\alpha} \quad , \tag{6.31}$$

$${}_0\overset{\alpha}{\mu}_k = \sum_{\beta=1}^{\nu} \left(\rho_\alpha \frac{\partial A'_\alpha}{\partial \rho_\beta} \frac{\partial \rho_\beta}{\partial x_k} - \rho_\beta \frac{\partial A'_\beta}{\partial \rho_\alpha} \frac{\partial \rho_\alpha}{\partial x_k} \right) \quad , \tag{6.32}$$

as found previously [6]. These results are appropriate for a mixture of viscous fluids.

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Constitutive equations are discussed for a mixture of any number of materials with elastic and viscous properties in which the constituents may have different temperatures.

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